Experimental Investigation of the Stability of the Laminar Supersonic Cone Wake

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An instability investigation was carried out in the wake of a 7° half angle, sharp cone suspended magnetically. Hot-wire fluctuation measurements were made in the wake for a range of Reynolds numbers and at a Mach number 4.3. These measurements indicated a completely stable near wake at $Re_{\infty d}$ = 51,600 and large amplification of small disturbances at $Re_{\infty d}$ = 61,900. The amplified waves were highly concentrated with respect to frequency, with a number of pronounced harmonics being present. Amplitude and phase measurements of the spectral components indicated that the instability process fits within the framework of linear stability theory as formulated by Gold, with each mode having a wavefront shaped like a circular helix. It appears that each succeeding mode has an additional thread in the helical wavefront. However, the measured fundamental oscillation has a larger amplification rate and a much lower frequency than predicted by Gold's theory. In addition, weak nonlinear interactions were observed, which grow stronger with downstream position.

Nomenclature

a = speed of sound
b = wake half width—[distance from axis to the point where $u = u_e - (u_e - u_{\mathbb{C}}/2.718)$]
C_g = group velocity
C_R = phase velocity in x direction
d = model base diameter (sometimes probe diameter)
e = hot-wire voltage
Δe = hot-wire sensitivity to flow quantity fluctuation
f = frequency
L^* = $\left[2\int_0^b \frac{T_e}{T} rdr\right]^{1/2}$, transformed wake width
M = Mach number
ΔM = $(u_e - u_{\mathbb{C}})/a_e$ difference Mach number
m = mass flux = ρu
$\Delta m_{\mathbb{C}}$ = $m_e - m_{\mathbb{C}}$
n = azimuthal mode number
p_o = stagnation pressure
q = a fluctuating quantity
Re = Reynolds number = $\rho(u/\mu)$ (length)
Re_δ = Reynolds number based on displacement thickness of wake and conditions just outside the wake
r = radial coordinate
r_c = critical radius (where $u = C_R$)
S = Strouhal number = fd/u
T = static temperature
T_o = stagnation temperature
ΔT = $T_{\mathbb{C}} - T_e/T_e$ temperature excess parameter
u = velocity in x direction
x = longitudinal coordinate
y = vertical coordinate
z = horizontal coordinate
α = wave number (axial)
α_o = one particular wave number
αC_I = temporal amplification rate
$\alpha' C_I'$ = $\alpha C_I b/(u_e - u_{\mathbb{C}})$ nondimensional temporal amplification rate
θ = azimuthal angle
ρ = density
η = $\left[2\int_o^r \frac{T_e}{T} rdr\right]^{1/2}$ transformed radial coordinate

Subscripts

$\mathbb{C}$ = centerline
e = edge of viscous region
rms = root-mean-square
∞ = freestream
$\langle\ \rangle$ = mean value
$(\ )'$ = fluctuating value

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Introduction

MEAN measurements made in the supersonic cone near wake at freestream Reynolds numbers of 40,600 and 94,300 showed fundamentally different decay properties in the velocity defect and temperature excess downstream of the rear stagnation point.[1] The velocity defect and temperature excess of the higher Reynolds number wake (turbulent after the recirculation region) decayed much more rapidly than in the fully laminar case. Simple hot-wire fluctuation measurements confirmed the obvious, viz, the wake is fully laminar at $Re_{\infty d}$ = 40,600 in the region of measurement, and transition to a turbulent wake occurs just downstream of the rear stagnation point at $Re_{\infty d}$ = 94,300.

In the case of the supersonic cone wake, the initial stage of the transition process— the laminar instability process— is very ordered. Figures 1 and 2 present an oscilloscope trace and a spectrum of the hot-wire voltage fluctuation with the probe positioned near the radius of maximum shear in the wake, six diameters downstream of the base. The fluctuations, which have a character similar to Tollmien-Schlichting instability waves, are self excited. The most striking feature of the fluctuations, besides the fact that they appear as discrete oscillations on the oscilloscope, is the existence of many discrete harmonics.

Numerous shear flows have been shown to have nonlinear regions with multiharmonic instability waves (see, i.e., Browand[2]). However, there exists strong evidence, both in theoretical and experimental work, which suggests that the fluctuations in the supersonic cone wake shown in Figs. 1 and 2 are still of infinitesimal size and could be described within the context of a complete linear theory.

In a series of publications, Gold[3–5] has reported his theoretical investigations of the stability of supersonic wakes—both two-dimensional and axisymmetric. He formulated the problem using inviscid laminar stability theory and Gaussian mean flow profiles (in the transformed Howarth-

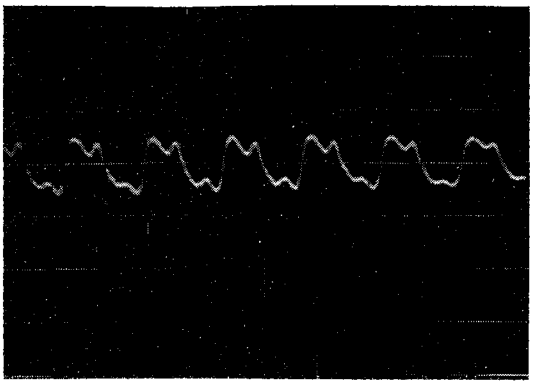

Fig. 1 Oscilloscope trace of instantaneous hot-wire voltage fluctuation at $Re_{\infty d}$ = 61,900 (sweep time equals 0.5 msec cm).

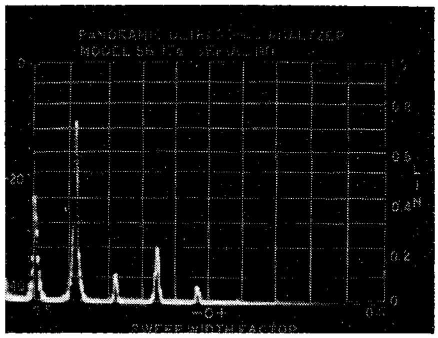

Fig. 2 Spectrum of hot-wire voltage fluctuation at $Re_{\infty d}$ = 61,900 (the peak at $f = 0$ is a timing mark and the sweep width = 10 kHz).

Dorodnitsyn Coordinate) for velocity and temperature. For axisymmetric wakes, a fundamental consideration of the instability process is the aximuthal dependence of the fluctuations. Gold points out that this dependence must be sinusoidal of the form $e^{in\theta}$, where n is an integer. (This is identical to the form for the incompressible axisymmetric wake stability studied by Batchelor and Gill.[6]) Any fluctuating quantity (such as velocity) can thus be written in the form $q(x,r,\theta,t) = \hat{q}(r) \exp[i\alpha(x - Ct) + in\theta]$ where waveforms are lines in which $(\alpha x + n\theta)$ is constant. These waveforms are circular helices, with n equal to the number of threads and with the orientation of the helix (left- or right-hand thread) given by the sign of n.

Two specific results of Gold's theoretical investigation are particularly important in light of the present experimental situation:

1) The importance of the temperature excess parameter is established. As a result of the application of Sturm's oscillation theorem, the number of possible modes of instability is seen to increase rapidly with the temperature excess for $\Delta T > 1$. This argument can be carried further by assuming isoenergetic mean flow, in which case the number of allowable modes increases with difference Mach number $\Delta M = (u_e - u_{℄})a_e$.

2) The familiar necessary and sufficient condition applies for the existence of neutral subsonic disturbances; namely that the gradient of the density-vorticity product vanishes for some critical radius. For the $n = 0$ mode, this results in $r_c = 0$, and hence, axisymmetric, neutral subsonic disturbances are impossible. This leads to the conclusion that the $n = 0$ mode will not be amplified.

For a fluctuation of multiharmonic content, such as measured in the supersonic cone wake (Figs. 1 and 2), the possibility exists that each spectral component corresponds to a different mode—each with a different azimuthal mode number. Figure 3 shows a number of wavefronts of possible modes of instability.

An important experimental discovery by Kendall[7] lends credence to the argument that the multiharmonic oscillations measured in the supersonic cone wake correspond to the different instability modes predicted by linear stability theory. Kendall found that for a Mach number 4.5 flat plate boundary layer the instability process is characterized by two main modes. [At lower Mach numbers only one mode exists (Laufer and Vrebalovich[8]).] He found that the first mode waves propagate in a direction 55° to the flow direction. The second-mode waves were found to be most highly amplified when excited as two-dimensional disturbances. The amplification rates measured by Kendall proved to be in good agreement with theoretical predictions by Mack[9] for oblique waves.

From the work of Kendall and Mack one would expect to find both higher modes that are amplified and also obliquely travelling waves in the cone wake. (In the axisymmetric case these are helical wavefronts.)

Experimental Objectives

The instability of the laminar supersonic cone wake was studied experimentally with the following specific objectives:

1) Establish the nature of the fluctuations encountered in the supersonic cone wake to determine whether or not the oscillations have a structure consistent with the predictions of linear stability theory. An additional test of the theory was to compare the growth of the waves with exponential behavior.

2) Establish the role of Reynolds number in the instability process.

3) Establish the effect on the instability process of small model inclinations, which have been shown to have a significant effect on the wake mean flow.[10]

4) Establish the extent of the nonlinear interactions by studying the intermodal behavior concurrent with the investigation of linear behavior.

From the mean flow investigation,[1] properties of the flowfield which are particularly important in the instability process are as follows. The velocity-defect-changes downstream of the rear stagnation point are moderate over a length equal to a wake diameter, but are very large over a wavelength. Because of the acceleration it is expected that numerical values from parallel flow stability theory would not agree with the present measurements. The difference Mach number across the viscous wake is about 2. This

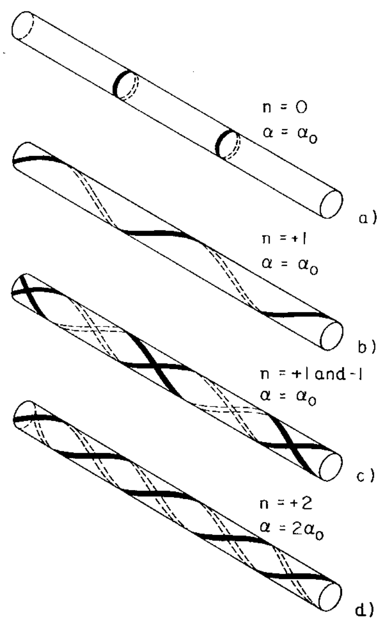

Fig. 3 Sketches of various instability modes.

Table 1 Summary of test conditions

p_o, psia	T_o, °F	T_w	$Re_{\infty d}$	$M_{\infty \text{ ave}}$	Model
15	250	adiabatic	51,600	4.31	7° half angle
18	250	adiabatic	61,900	4.30	sharp
22	250	adiabatic	77,000	4.28	cone

indicates that waves travelling obliquely to the flow direction will likely be subsonic with respect to both inner and outer streams. Finally the temperature excess parameter is of the order of 3, high enough to indicate the presence of multimodes of instability.[5]

Description of the Experiment

Experimental Equipment

The models used in the fluctuation experiments were the same 7° half-angle cones for which the mean flow properties of the near wake were established.[1] Also, the magnetic suspension system performance was described in McLaughlin.[11] In the case of this investigation it was necessary to run at Reynolds numbers low enough such that the boundary layers on the walls of the wind tunnel were laminar so as to minimize residual noise.

A summary of the test conditions is given in Table 1.

The hot-wires were constructed in the same manner as described in Browand et al.[10] The sensor aspect ratios were all well over 200. We can conclude from Behren's work in measuring fluctuations with hot wires of various aspect ratios[12] that end effects do not introduce significant errors in our experiments. Examination of spectra of the hot-wire signal located in the broad band disturbance region of the wind-tunnel jet edge showed no peaks, indicating that wire vibration was not a problem.

The hot-wires were operated with a Shapiro-Edwards Model 50B constant current system. This standard set has an a.c. amplifier that is frequency compensated to cancel the inherent lag effect of the hot-wire caused by thermal inertia. In measurements involving two probes (phase measurements) the signal from the second probe (probe B) was amplified with a Tektronix Model 122 low level preamplifier, since there was only one compensated amplifier in our system. Naturally, none of the probe B data was used in amplitude measurements because of the roll-off with frequency of the hot-wire response. The Tektronix amplifier does not change the phase in the same way as does the Shapiro-Edwards amplifier and therefore in measuring phase difference, the shifts in the two amplifiers must be accounted for.

Most of the hot-wire voltage fluctuation data was tape recorded and analyzed from the recordings. Because of the multiharmonic content of the oscillations, on line data reduction would not have been economically prudent. Two tape recorders were used. The first, an Ampex Series FR-100A Recorder/Reproducer with 14 channels, was limited to a maximum frequency response of 5000 Hz. The second was a Roberts Model 1740X tape deck with 2 channels and a maximum frequency response of 20,000 Hz.

The tape recordings were analyzed using a Spectral Dynamics Corp. Transfer Function Phase Ratio Measurements System. This system contained two SD101A Dynamic Analysers with 100-Hz band width crystal filters, an SD104A-S Sweep Oscillator used for tuning the analyzers and an SD112 Voltmeter/Log Converter which links in a feedback path with the wave analyzers. As a result, by examining the two filtered signal outputs, one can measure the phase difference between the tuned spectral components of the input signals.

In addition to the Spectral Dynamics wave analyzers an ultrasonic spectrum analyzer model SD15a made by Panoramic Radio Products Inc. was used for instantaneous examination of the spectrum of a given wave. This instrument displays the spectrum on a cathode ray tube which was monitored constantly during the wind-tunnel runs. Photographs of the display screen provide a permanent record of the spectra. (See Fig. 2.)

Experimental Procedures

Phase measurements

The two-probe technique was used for all the phase measurements. The fluctuating voltages from each of the hot-wires were simultaneously recorded on two channels of the Ampex tape recorder. Two other channels were used for probe position and voice.

The phase differences between the spectral components of the two recorded signals were measured in the following manner. The tape recorder outputs were input to the two-wave analyzers. The analyzers were set up so that one was the slave of the other, meaning their tuning signals were phase locked. One oscillator was used to tune both analyzers, and this was set to each spectral component in turn. The filtered signal outputs were displayed on an oscilloscope, and the phase was measured from photographs of the signals. The measured phase was corrected for the phase shifts encountered in the amplifiers. In doing this, the assumption was made that each probe had the same time constant. Measurement of the time constants of both wires, using the square wave technique, demonstrated that the error caused by this approximation was small.

The phase of each spectral component of the oscillation was highly dependent on radial location in the wake. Changes of 180° over 0.02 in. change in radius were not uncommon. To ensure that the two probes were at the same r location, the $r = \infty$ (in boundary-layer coordinates) was used as the set position. It was assumed that as the outer edge of the regions of high shear were reached the phase would asymptote to a constant value as the amplitude symptotes to zero. All phase measurements were made at an axial location 1-diam downstream of the model.

There exists a complication concerning the practical side of measuring the phase of the spectral component. Mathematically speaking, the full complement of azimuthal mode numbers includes negative integers as well as the positive ones. In physical terms this means wavefronts that form a left-hand helix as well as those which form a right-hand helix (see sketches in Fig. 3). For a given frequency the stationary hot-wire is not capable of separating the positive from the negative integer azimuthal wave number modes. Since the oscillations are small they will add linearly, and the hot-wire will sense an aggregate wave whose amplitude will be a combination of the amplitudes of two modes and whose phase will be somewhere between the phase of the two (depending on their relative amplitudes).

Since the amplitudes of the positive and negative modes will, at any instant, depend upon the initial conditions (which are random disturbances), we should expect to measure random phase angles. Random phase angles will not result if a helix of only one orientation is present. This was the case in the measurements of instability waves in incompressible axisymmetric wakes by Sato and Okada[13] and by Kendall.[14] Sato and Okada found only the right-hand helices in the wake of a slender body. Kendall's measurements in the wake of a sphere indicated waves with a preferred orientation for $Re_{\infty d} \approx 2000$. However, at $Re_{\infty d} \approx 500$ he found a helical pattern whose direction reversed at irregular intervals of a few cycles.

Amplitude measurements

One probe (probe A) was used in the amplitude measurements, and the fluctuating voltage signal was recorded on one channel of the Roberts tape deck. Continuous traverses were taken across the wake for 11 axial locations from 1–6 diam behind the base of the model.

The time constant of the hot-wire was determined for the conditions just outside the viscous core, and the amplifier was compensated accordingly. In general, the calculation of the fluctuating flow quantities from the measurement of hot-wire voltage fluctuation is rather involved.[15] A reasonable assumption (used by Behrens[12]) that the hot-wire responds basically to mass flux fluctuations was used in the present data reduction, i.e., $e' = -\Delta e_m[(\rho u)'/\langle \rho u\rangle]$. The sensitivity coefficient Δe_m was determined from mean flow data (for details see Ref. 11).

Experimental Results

Effect on Spectra of Instability Waves

In many unbounded flows, such as wakes, the Reynolds number is relegated to a somewhat secondary role in the instability process. In the inviscid formulation,[5] the Reynolds number does not enter explicitly, but its effect is felt through the changes in the mean flow profile.

In this particular flow situation the Reynolds number does not appear to play such a secondary role. Figure 4 presents a spectrum taken at $x/d = 5$ at a freestream Reynolds number of 51,600. Notice that the waves do not have the discrete harmonic content of the $Re_{\infty d} = 61{,}900$ case (Fig. 2). In addition, the level of fluctuations in the lower $Re_{\infty d}$ case is less than for $Re_{\infty d} = 61{,}900$ by a factor of about 100 at 5 diam behind the model. It appears that reduction in the Reynolds number of only 20% completely stabilizes the wake within the region of measurement.

Measurements at higher Reynolds number were not performed in any detail. The reason for this was the fact that the wind-tunnel boundary layers underwent transition at a freestream Reynolds number only 20% higher than at $Re_{\infty d} = 61{,}900$ when the waves initially appear.

The spectra at various axial locations in the high shear radius of the wake at $Re_{\infty d} = 61{,}900$ have the same multiharmonic behavior illustrated in Fig. 2. The Strouhal number corresponding to the fundamental spectral component is $S = fd/U_\infty = 0.028$, which is in close agreement with the value of 0.017 measured by Kendall[16] in the wake of a sphere at $M = 3.7$. However, it is much smaller than the value of $S = 0.14$ measured by Sato and Okada[13] in the incompressible axisymmetric wake. Correspondingly, it is expected that the wavelengths of the present instability waves are much longer than previous experience with waves in incompressible shear flows.

Phase Measurements

Figure 5 shows the aggregate of phase measurements of the first three modes. Each point on the graphs represents a single phase measurement taken from the oscilloscope traces of the filtered signal outputs. An important point is that

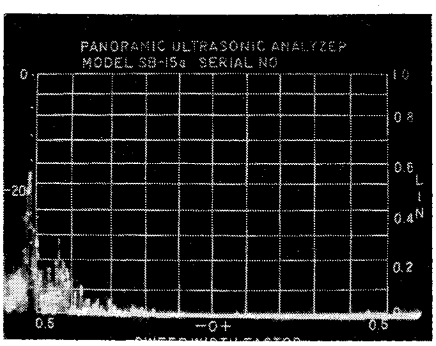

Fig. 4 Spectrum at $x/d = 5$, $Re_{\infty d} = 51{,}600$ (sweep width = 10 kHz).

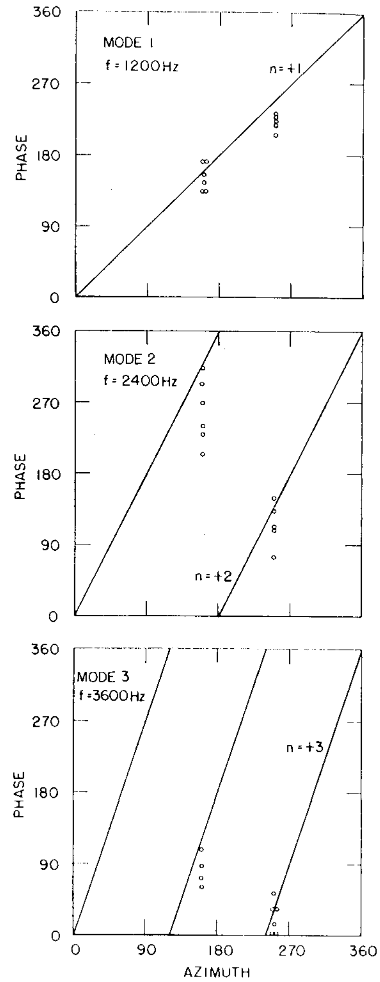

Fig. 5 Results of phase measurements of the first 3 modes.

in the tape recording sequence, both probes were repositioned between each measurement. They were both driven from the "extremes" and repositioned to the radial location at which the signals were barely distinguishable. This was done to check repeatability. The idea is that all random noise effects will "average out" and leave a dependable measurement.

All phase measurements at the azimuthal position of 70° were discarded because they were judged unreliable. For a phase measurement at this azimuthal position the scanner probe splits the wake, and with the reference probe also in the wake, probe interference is unacceptably large. The mean flow was distorted enough to grossly affect the radial distribution of fluctuation amplitude, making it different than measured under single probe conditions. This problem did not occur at azimuthal positions of 160° and 250°.

In general, agreement is good with azimuthal dependences of the first three spectral components like $e^{in\theta}$ (n = 1,2, and 3). Gold's prediction (for a Gaussian mean flow profile) that the $n = 0$ (axisymmetric) mode will not be amplified is confirmed in the present experiments. It also appears that the left-hand helices (negative mode numbers) are not present. This could be caused by some residual nonuniformity in the freestream flow. None of the phase measurements indicated a shifting of the orientation of the disturbance helices from right-hand to left-hand pitch (from $n = +1$ to $n = -1$, etc.).

Fluctuation Amplitude Measurements

Figure 6 presents profiles of mean square hot-wire voltage fluctuation at various axial positions behind the model. These profiles (containing fluctuations of all frequencies) are similar to the results of other experimenters in various wake flows.[12,17] There appears, however, to be a distinct

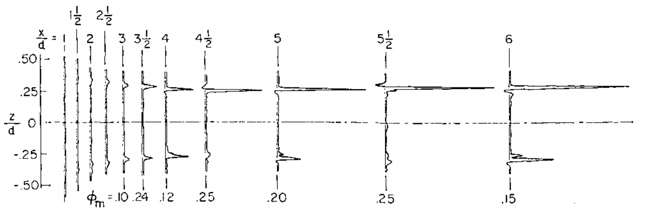

Fig. 6 Mean square full wave fluctuation measurements at $Re_{\infty d}$ = 61,900.

asymmetry in a number of the profiles, and experiments indicate that this asymmetry is caused by slight model inclination.

Extensive Pitot pressure measurements at small model inclinations[10] indicated a substantial qualitative as well as quantitative change in the flowfield of the cone near wake. It seems likely that this would manifest itself in corresponding changes in the instability process. To investigate this, fluctuation measurements were made in the wake of a cone at $-3°$ angle of attack. Figure 7 presents mean square hot-wire voltage fluctuation profiles along with the mean voltage profiles for 0 and $-3°$ model angle of attack. The traverses are made in the vertical direction, in the plane of the model angulation. The most predominant effect is the almost complete disappearance of the peak of fluctuating voltage on the leeward side of the angulated cone. This decrease in fluctuation level was not nearly matched by the slight increase in level on the windward side (less than 20% at this model inclination). Another change which was evident was the generation of oscillations at frequencies other than the harmonics.

The problem of setting the model to absolutely zero inclination with respect to the stream was discussed in Ref. 11. Included in Fig. 6 with each profile is an estimate of the model angle of yaw (ϕ_m) for that particular traverse. Because the uncertainty in this measurement is about $\pm 0.1°$ it is included only as a qualitative indicator.

It seems reasonable that if $-3°$ model angulation can make the peak in fluctuations almost disappear from the leeward side, then 0.2° ought to substantially affect the level there. Also, if $-3°$ changes the fluctuations in the windward side by less than 20%, then 0.2° should change it very little. Assuming this is to be the case (a $0.2 \pm 0.1°$ model yawing) only data from the windward side is used when examining the growth of fluctuations.

Figure 8 presents a plot of the peak amplitude of the first four spectral components of the instability waves. As discussed earlier, only the windward side of the small model inclination data is used. The fluctuation amplitudes are nondimensionalized with the mass flux defect $\Delta m_{\text{℄}} = m_e - m_{\text{℄}}$, so that the changes in the mean flow are essentially accounted for. To construct such a plot, the radial distribution of each spectral component is first plotted for each axial location and the peak values are selected from the plots. As expected, the peaks of the different modes do not occur at the same radial location (see Ref. 11 for details).

The axial distributions of amplitude have the general appearance of exponentials. To investigate this more thoroughly the same plots are made on semilog paper (Fig. 9). In general, the plots are straight lines indicating the exponential nature of the growth process. The largest departures from exponential growth occur upstream of x/d = 3. This is not surprising in view of the convergence of the streamlines near that point. (The rear stagnation point is located at about $x/d = 2.5$.) From there downstream the flow is parallel to order $1/(\alpha Re_\delta)$, and exponential growths are expected. At the furthest downstream positions a noticeable decrease in the growth rate of the first mode is evident (a deviation from exponential growth). The following evidence suggests that this is the start of the nonlinear region. First, the decrease in growth rate of mode 1 is also present in measurements in an incompressible wake by Sato and Kuriki[18] in the initial stages of the nonlinear region. Second, the fluctuation amplitude of the first mode at this location is almost 2% ($m_{rms}'/\Delta m_{\text{℄}}$) which compares favorably with the linearity limits ranging from 2 to 5% (tabulated by Browand[2]) for a wide class of incompressible flows. Finally the results of amplitude modulation experiments indicate significant interactions between the modes at these furthest downstream locations, indicating a weak nonlinear behavior (see later discussion).

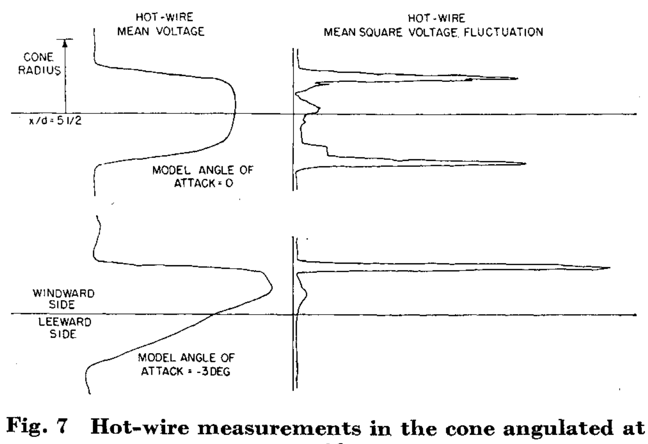

Fig. 7 Hot-wire measurements in the cone angulated at $-3°$.

Growth rate

The slopes of the plots of the first four modes look to be identical, indicating similar growth rates. From this slope the spatial rate of amplification is

$$\frac{1}{m'/\Delta m_{\text{℄}}}\,\frac{d}{d(x/d)}\left[\frac{m'}{\Delta m_{\text{℄}}}\right] = \frac{d}{d(x/d)}\left[\ln\left(\frac{m'}{\Delta m_{\text{℄}}}\right)\right] = 1.11$$

From linear theory we know that this is equal to $\alpha C_I d/C_g$ with all quantities in dimensional terms. Since the wave speeds of the modes were not measured, we must assume a value for the group velocity. If we assume C_g equals the wave speed of mode 1 and use the calculated wave speed of Gold,[5] we obtain $C_g = 0.85\ u_e$ for mean flow parameters $\Delta M = 2$, $\Delta T = 3$ and $u_e - u_{\text{℄}} = 0.6\ u_e$, which represent the values measured at $x/d = 4$. We introduce the half-wake

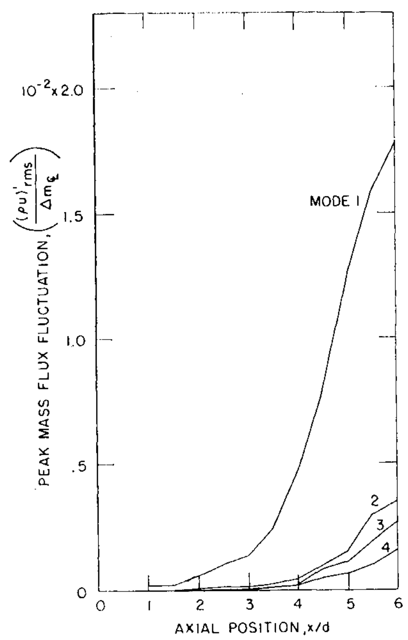

Fig. 8 Axial distribution of peak amplitudes of the first 4 modes.

width b, as the distance from the axis to the point where the velocity equals $u_e - (u_e - u_{\mathbb{C}})/2.718$; for our mean flow $b/d \approx 0.25$ from $x/d = 3$ to 6. Finally the nondimensional temporal amplification rate $\alpha' C_I' = \alpha C_I b/(u_e - u_{\mathbb{C}})$ is calculated to be 0.4 from the measured spatial amplification rate. This growth rate is much larger than the value of $\alpha' C_I' = 0.05$ measured by Sato and Okada in the incompressible wake.[13] It is also considerably larger than the peak value of $\alpha' C_I' = 0.085$ predicted by Gold's compressible inviscid stability theory[5] for the present mean flow.†

In addition to the discrepancy in the amplification rate between our measurements and Gold's theory, comparison of the mode 1 frequency indicates the measured fundamental frequency to be about 16 times smaller than the theoretically predicted frequency for maximum amplification. (We saw earlier that the frequency comparison with the incompressible wake indicated a similar difference.[13])

There appears to be three possible reasons for the discrepancy between the theory and the present measurements.

1) The theory used Gaussian mean flow profiles, whereas the measured profiles are closer to the "top hat" shape (plotted vs the transformed Howarth-Dorodnitsyn coordinate η).[1] Gold made additional calculations[5] which show that this profile can account for a factor of three in growth rate.

2) The theory neglects the viscous terms in the stability equations. Kendall[17] suggests that for the difference Mach number $\Delta M \approx 2$ viscosity plays an important role. The great difference between the fluctuations in the wake at $Re_{\infty d} = 51{,}600$ and $Re_{\infty d} = 61{,}900$, discussed earlier, seems to substantiate the importance of the viscosity in the instability process.

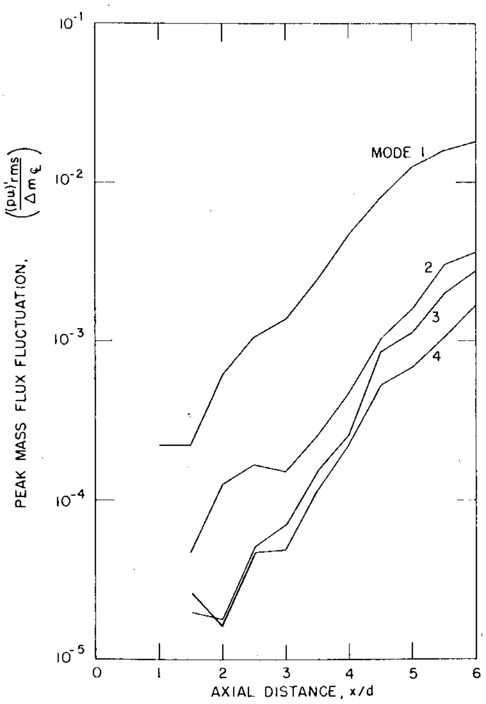

Fig. 9 Axial distribution of the log of the peak amplitudes of the first 4 modes.

† Care must be taken to change the numerical values from Gold[5] and Sato and Okada[13] to account for different characteristic lengths and velocities. In particular Gold uses a transformed wake width

$$L^* = \left[2\int_0^b \frac{T_e}{T}\, rdr\right]^{1/2}$$

Ref. 11 stands in error because it overlooked this point.

a) no correlation

b) correlation

c) anticorrelation

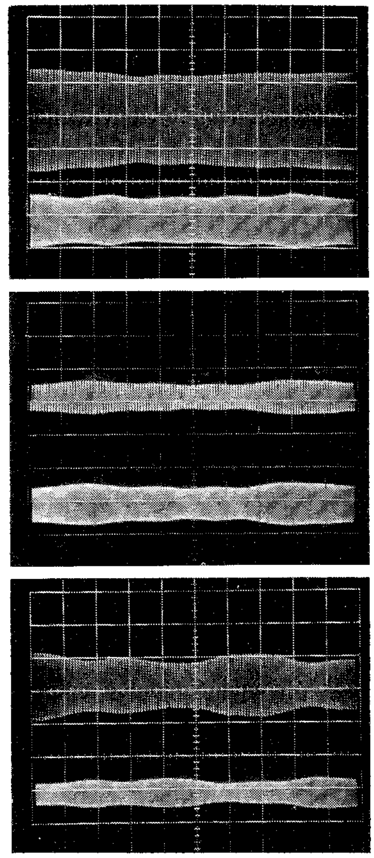

Fig. 10 Instantaneous filtered signal outputs of two spectral components for correlation of intermittency.

3) The theory neglects the derivatives of the mean flow quantities in the x direction in the stability equations. This quasi-parallel flow formulation is usually accurate when the accelerations are not great. In this mean flow situation the velocity on the centerline increases by over 20% in one wake thickness downstream of the neck. Since a wavelength is many times the wake diameter, the mean velocity change in a wavelength is very great. This effect, if included in the theory, would undoubtedly affect resulting amplification rates.

Modulation Measurements

Modulation measurements were performed by comparing the instantaneous amplitudes of two of the spectral components of one hot-wire signal. The filtered signal outputs were displayed simultaneously on the oscilloscope and long sweep times were used. The modulation of the amplitude of the envelopes of the two modes was examined for correlation or anticorrelation.

Over sixty oscilloscope traces were recorded in the modulation investigation. The envelope of mode 1 was compared with that of either mode 2 or 3. Many of the traces indicated that the amplitudes of the two modes were modulated independently. Figure 10a shows such a typical trace where the amplitude modulation of mode 1 is not at all correlated to that of mode 2. As the probe moved further downstream more and more traces indicated either a correlation or an anticorrelation in the modulation of the amplitude of the two modes. Figure 10b, c shows typical traces of these two cases. In terms of the records taken, no correlations were noticed upstream of 3-model diam, whereas 20% of the traces at 4 diam and fully 60% of the traces at 6 diam were either correlated or anticorrelated in amplitude modulation.

Although not enough experiments of this type were performed to be conclusive about the findings, there appears to be substantial evidence of weak nonlinear interaction in the cone wake. Particularly inside the critical layer, energy exchange between the modes appears to take place, as shown by the anticorrelations in amplitude modulation. Correlation of some amplitude modulation suggests that some of the energy of mode 3 for example has transferred from mode 1. As is to be expected, the intermodal correlations are more prevalent at the downstream positions where the amplitudes of all fluctuations are larger.

Summary and Conclusions

Detailed measurements indicate that the structure of the fluctuations fit within the framework of laminar stability theory. The exponential growth and the azimuthal dependences of the multimodes agree with the linear formulation of Gold[5] for axisymmetric wakes. In particular, it appears that the axisymmetric mode is not amplified and each mode has a wavefront shaped like a circular helix with each succeeding mode having an additional thread intertwined in the other helices. However, the $n = 1$ mode instability wave measured in the present experiments has a much lower frequency and higher growth rate than predicted by Gold's theory.

Comparison of the modulation of two spectral components of the same hot-wire signal indicated the presence of weak nonlinear interaction. This interaction becomes stronger as the amplitudes of the waves grow.

References

[1] McLaughlin, D. K. et al., "Experimental Investigation of the Mean Flow of the Laminar Supersonic Cone Wake," *AIAA Journal*, Vol. 9, No. 3, March 1971, pp. 479–484.

[2] Browand, F. K., "An Experimental Investigation of the Instability of the Incompressible Separated Shear Layer," *Journal of Fluid Mechanics*, Vol. 26, Pt. 2, 1966, pp. 281–307.

[3] Gold, H., "Stability of Laminar Wakes," Ph.D. thesis, 1963, California Institute of Technology, Pasadena, Calif.

[4] Lees, L. and Gold, H., "Stability of Laminar Boundary Layers and Wakes at Hypersonic Speeds Part I Stability of Laminar Wakes," *International Symposium on Fundamental Phenomena in Hypersonic Flow*, edited by J. G. Hall, Cornell University Press, Ithaca, N.Y., 1966, pp. 310–331.

[5] Gold, H. "Stability of Axisymmetric Laminar Wakes," AIAA Paper, Williamsburg and Hampton, Va., 1964.

[6] Batchelor, G. K. and Gill, A. E., "Analysis of the Stability of Axisymmetric Jets," *Journal of Fluid Mechanics*, Vol. 14, Pt. 4, 1962, pp. 529–551.

[7] Kendall, J. M., Jr., "Supersonic Boundary Layer Stability Experiments," *Proceedings of the Boundary Layer Transition Study Group Meeting*, Vol. II, Aerospace Rept. TR-0158, (S3816-63)-1, 1967.

[8] Laufer J. and Vrebalovich T., "Stability and Transition of a Supersonic Laminar Boundary Layer on an Insulated Flat Plate," *Journal of Fluid Mechanics*, Vol. 9, No. 2, Oct. 1960, pp. 257–299.

[9] Mack, L. M., "Viscous and Inviscid Amplification Rates of Two and Three-Dimensional Disturbances in a Compressible Boundary Layer," *JPL Space Program Summary 37-42*, Vol. IV, 1966, Pasadena, Calif., pp. 158–162.

[10] Browand, F. K., Finston, M., and McLaughlin, D. K., "Wake Measurements Behind a Cone Suspended Magnetically in a Mach Number 4.3 Stream," CP 19, May 1967, AGARD.

[11] McLaughlin, D. K., "Experimental Investigation of the Mean Flow and Stability of the Laminar Supersonic Cone Wake," Ph.D. thesis, 1969, Massachusetts Institue of Technology, Cambridge, Mass.

[12] Behrens, W., "Far Wake Behind Cylinders at Hypersonic Speeds: II Stability," *AIAA Journal*, Vol. 6, No. 2, Feb. 1968, pp. 225–232.

[13] Sato, H. and Okada, O., "The Stability and Transition of an Axisymmetric Wake," *Journal of Fluid Mechanics*, Vol. 26, Pt. 2, 1966, pp. 237–253.

[14] Kendall, J. M., Jr., "The Periodic Wake of a Sphere," *JPL Space Programs Summary No. 37-25*, Vol. IV, 1963, Pasadena, Calif., pp. 251–252.

[15] Morkovin, M. V., "Fluctuations and Hot-Wire Anemometry in Compressible Flows," AGARDograph No. 24, Nov. 1956.

[16] Kendall, J. M., Jr., "Experimental Study of Cylinder and Sphere Wakes at a Mach Number of 3.7," Technical Rept. 32–363, Nov. 1962, JPL, Pasadena, Calif.

[17] Demetriades, A., "Some Hot-Wire Anemometer Measurements in a Hypersonic Wake," *Proceedings of the 1961 Heat Transfer and Fluid Mechanics Institute*,

[18] Sato, H. and Kuriki, K., "The Mechanism of Transition in the Wake of a Thin Flat Plate Placed Parallel to a Uniform Flow," *Journal of Fluid Mechanics*, Vol. 11, No. 3, 1961, pp. 321–352.